Griffiths

9.3: Show that $Tr(\lambda^\alpha \lambda^\beta) = 2\delta^{\alpha\beta}$

For diagonal terms, it's obvious each λ matrix when squared has trace $\neq 2$.

For the off diagonal terms, first notice that the trace vanishes for product of a symmetric and antisymmetric matrix: suppose $g_{\alpha\beta}$ is symmetric, $\gamma^{\alpha\beta}$ is antisymmetric, then

$$Tr(g\gamma) = g_{\alpha\beta}\gamma^{\beta\alpha} = -g_{\alpha\beta}\gamma^{\alpha\beta} = -g_{\beta\alpha}\gamma^{\alpha\beta}$$
$$\Rightarrow Tr(g\gamma) = 0.$$

By explicit computation, one can show that the product of 2 symmetric λ's and the product 2 antisymmetric λs have their trace vanish unless they are equal:

$\lambda_1\lambda_3 = \begin{pmatrix} 0 & -1 & \\ i & 0 & \\ & & 0 \end{pmatrix}$, $\lambda_3\lambda_4 = \begin{pmatrix} & 0 & 1 \\ 0 & & \\ & & \end{pmatrix}$, $\lambda^4\lambda^3 = \begin{pmatrix} i & & \\ & 0 & \\ & & -i \end{pmatrix}$, $\lambda_5\lambda_6 = \begin{pmatrix} 0 & -i & 0 \\ & 0 & \\ & & 0 \end{pmatrix}$,

$\lambda_1\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ & & 0 \end{pmatrix}$, $\lambda_3\lambda_5 = \begin{pmatrix} & & -i \\ & 0 & \\ 0 & & \end{pmatrix}$, $\lambda_4\lambda_6 = \begin{pmatrix} 0 & 1 & 0 \\ & 0 & \\ & & 0 \end{pmatrix}$, $\lambda_5\lambda_8 = \begin{pmatrix} & & 2i \\ & 0 & \\ i & & \end{pmatrix}\frac{1}{\sqrt{3}}$,

$\lambda_1\lambda_5 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & & 0 \end{pmatrix}$, $\lambda_3\lambda_6 = \begin{pmatrix} 0 & & \\ & 0 & -1 \\ & & 0 \end{pmatrix}$, $\lambda_4\lambda_8 = \begin{pmatrix} & & -2 \\ & 0 & \\ 1 & & \end{pmatrix}\frac{1}{\sqrt{3}}$, $\lambda_6\lambda_8 = \begin{pmatrix} 0 & 0 & \\ 0 & 0 & -2 \\ 0 & -2 & 0 \end{pmatrix}\frac{1}{\sqrt{3}}$.

$\lambda_1\lambda_6 = \begin{pmatrix} & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$,

$\lambda_3\lambda_8 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}\frac{1}{\sqrt{3}}$,

$\lambda_1\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} & 1 & \\ 1 & & \\ & & 0 \end{pmatrix}$,

$\lambda_2\lambda_5 = \begin{pmatrix} 0 & 0 & \\ 0 & 0 & 1 \\ & & 0 \end{pmatrix}$, $\lambda_5\lambda_7 = \begin{pmatrix} & 1 & 0 \\ 0 & & 0 \\ & & 0 \end{pmatrix}$,

$\lambda_2\lambda_7 = \begin{pmatrix} 0 & 0 & \\ 0 & 0 & -1 \\ 0 & 0 & \end{pmatrix}$,